Asymptotic Behavior of Rotating Rarefied Gases with Evaporation and Condensation

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Abstract. The asymptotic behavior of the cylindrical Couette flow problem for a rarefied rotating gas with evaporation and condensation is studied when the state of equilibrium is perturbed by the following small thermodynamic forces: (i) the pressure difference of the evaporating gas; (ii) the angular velocity difference of the cylinders; and (iii) the temperature difference of the cylinders. The problem is solved by using the hydrodynamic equations that follow from the balance equations of mass, momentum and energy of a viscous heat conducting rarefied gas. The hydrodynamic equations are solved analytically by considering slip and jump boundary conditions. The fields of density, velocity, temperature, heat flux vector and viscous stress tensor are calculated as functions of the Knudsen number for each thermodynamic force and for different values of the angular velocity. The asymptotic behavior of these fields are compared with those obtained from the kinetic equation.

I INTRODUCTION

One of the basic principles of continuum mechanics is the principle of material frame indifference which states that the constitutive equations must be the same in inertial as well as in non-inertial systems of reference. This principle has no support within the framework of kinetic theory of gases as was pointed out by several authors, among others we cite Müller [1], Söderholm [2], Sharipov and Kremer [3] and Biscari and Cercignani [4].

A very simple non-inertial system is represented by the cylindrical Couette flow where a gas is confined between two rotating cylinders. The problems where the surfaces of the cylinders are rotating with different angular velocities and are at different temperatures were analyzed in the papers [3]. It was found that the anisotropy created by the rotation of the gas induces the following effects: i) a radial temperature gradient causes both radial and tangential heat fluxes; ii) the diagonal components of the viscous stress tensor are not zero but depend on the rotation frequency. These phenomena are observed not only in weak but also in strong non-equilibrium Couette flow.

The above mentioned papers consider the heat and momentum transfer only. To take into account the mass transfer we have to assume that the coaxial cylinders can absorb and emit particles of the fluid. One of the mechanisms of the mass exchange between the gas and the cylinders could be the evaporation and condensation of the particles on the cylinder walls. Gas flows with evaporation and condensation on the boundary have a very complicated solution even in systems at rest, i.e. without rotation. This type flow was investigated by many authors on the basis of the kinetic theory and an extensive list of the corresponding publications can be found in the paper [5].

The flow becomes more complex if the cylinders - that confines a gas in which processes of evaporation and condensation are considered - rotate. This problem was solved recently by us [6] in the whole range of the Knudsen number by assuming that the cylinders have the same temperature and rotate with the same velocity, but they evaporate the gas with weakly different pressures. Some analytical results on the Couette flow with evaporation and condensation in the continuum regime can be found in the paper [7], while interesting results on the non-linear Couette flow with evaporation and condensation are presented in the paper [8] where it was pointed out that a bifurcation of the flow is possible.

Recently [9] the transport phenomena in rotating rarefied gases that undergo evaporation and condensation on their surfaces were investigated by us on the basis of the kinetic theory of gases. We have considered

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14. ABSTRACT
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that the state of equilibrium was perturbed by the following small thermodynamic forces: (i) the pressure difference of the evaporate gas; (ii) the angular velocity difference of the cylinders; and (iii) the temperature difference of the cylinders. For every thermodynamic force all transport phenomena, of mass, momentum and heat transfer were calculated, i.e. the direct effects such as: (i) the mass transfer caused by the pressure difference, (ii) the momentum transfer caused by the velocity difference, and (iii) the heat flux caused by the temperature difference. Further the Onsager-Casimir reciprocity relations for the cross effects were verified. The used kinetic equation was the Shakov model of the Boltzmann equation and it has been solved by the discrete velocity method where the discontinuity of the distribution function has been taken into account.

The aim of this work is to find the asymptotic behavior of the above mentioned phenomena by using the hydrodynamic equations that follow from the balance equations of mass, momentum and energy of a viscous heat conducting rarefied gas. The hydrodynamic equations are solved analytically by considering slip and jump boundary conditions [10]. The fields of density, velocity, temperature, heat flux vector and viscous stress tensor are calculated as functions of the Knudsen number for each thermodynamic force and for different values of the angular velocity. The asymptotic behavior of these fields are compared with those obtained from the kinetic equation [9].

II HYDRODYNAMIC SOLUTION

We consider a rarefied gas between two coaxial rotating cylinders with radii R_0 and R_1 ($R_0 > R_1$). The cylinders are rotating around the z-axis and they are supposed to be so long that end effects can be neglected. Further we assume that the equilibrium state is weakly disturbed by the following three factors: (i) The outer cylinder evaporates the particles with an equilibrium pressure $P_0(R_0)$, while the pressure of evaporated particles from the inner cylinder P_1 slightly differs from the equilibrium pressure at its surface, i.e.

$$P_1 = P_0(R_1) + \Delta P, \qquad \frac{|\Delta P|}{P_0(R_1)} \ll 1. \tag{1}$$

The equilibrium pressure is given by the equation of state $P_0(r') = n_0(r')kT_0$ where $r' = \sqrt{x^2 + y^2}$ is the radial coordinate and $n_0(r')$ is the particle number density between the cylinders

$$n_0(r') = \frac{n_{00}(1 - R_1^2/R_0^2)(\beta\Omega_0 R_0)^2}{\exp[(\beta\Omega_0 R_0)^2] - \exp[(\beta\Omega_0 R_1)^2]} \exp[(\beta\Omega_0 r')^2], \qquad \beta = \left(\frac{m}{2kT_0}\right)^{1/2}. \tag{2}$$

In the above equation n_{00} denotes the number density when the cylinders are at rest ($\Omega_0 = 0$), m is the mass of a gas particle and k is the Boltzmann constant; (ii) The outer cylinder rotates with an angular velocity Ω_0, while the angular velocity of the inner cylinder Ω_1 slightly differs from Ω_0, i.e.

$$\Omega_1 = \Omega_0 + \Delta\Omega, \qquad \beta R_1|\Delta\Omega| \ll 1. \tag{3}$$

The smallness of $\Delta\Omega$ means that the difference between the velocities of the surfaces is smaller than the sound speed; (iii) The outer cylinder is at the equilibrium temperature T_0, while the temperature of the inner cylinder T_1 slightly differs from T_0, i.e.

$$T_1 = T_0 + \Delta T, \qquad \frac{|\Delta T|}{T_0} \ll 1. \tag{4}$$

There are two main parameters that determine the solution of the problem: the inverse Knudsen number which is the rarefaction parameter $\delta = \sqrt{\pi}R_0/(2\lambda_{00})$ where λ_{00} is the molecular mean free path at the density n_{00} and at temperature T_0; and the dimensionless angular velocity $\omega = \beta\Omega_0 R_0$.

In order to solve the above mentioned problem in the hydrodynamic regime flow we use the balance equations of mass density ϱ, momentum density $\varrho u_i'$ and internal energy density $\varrho\epsilon$ that read

$$\frac{\partial\varrho}{\partial t} + \frac{\partial\varrho u_i'}{\partial x_i'} = 0, \quad \frac{\partial\varrho u_i'}{\partial t} + \frac{\partial}{\partial x_j'}(\varrho u_i' u_j' + P\delta_{ij} - \sigma_{ij}') = 0, \quad \frac{\partial\varrho\epsilon}{\partial t} + \frac{\partial}{\partial x_i'}(\varrho\epsilon u_i' + q_i') + P\frac{\partial u_i'}{\partial x_i'} - \sigma_{ij}'\frac{\partial u_i'}{\partial x_j'} = 0. \tag{5}$$

These balance equations are supplemented by the constitutive equations for the viscous stress σ_{ij}', heat flux vector q_i' and specific internal energy ϵ:

$$\sigma'_{ij} = \mu \left[\frac{\partial u'_i}{\partial x'_j} + \frac{\partial u'_j}{\partial x'_i} - \frac{2}{3} \frac{\partial u'_r}{\partial x'_r} \delta_{ij} \right], \qquad q'_i = -\kappa \frac{\partial T}{\partial x'_i}, \qquad \epsilon = \frac{3k}{2m}. \tag{6}$$

The above equations represent the constitutive equations of a Newtonian heat conducting fluid without bulk viscosity. The coefficient of thermal conductivity κ is given in terms of the coefficient of shear viscosity μ by $\kappa = 15k\mu/(4m)$. For a monatomic ideal gas κ and μ are only functions of the temperature T.

By taking into account that all fields depend only on the radial coordinate r' and that we are looking for a stationary solution of the gas flow, Eqs (5) reduce to

$$\frac{1}{r'} \frac{\partial}{\partial r'} \left(\frac{P}{T} r' u'_r \right) = 0. \qquad \varrho \left(u'_r \frac{\partial u'_r}{\partial r'} - \frac{u'^2_\varphi}{r'} \right) = -\frac{\partial P}{\partial r'} + \frac{4}{3} \frac{\partial}{\partial r'} \left[\frac{\mu}{r'} \frac{\partial (r' u'_r)}{\partial r'} \right] - \frac{2}{r'} u'_r \frac{\partial \mu}{\partial r'}, \tag{7}$$

$$\varrho \left(u'_r \frac{\partial u'_\varphi}{\partial r'} + \frac{u'_r u'_\varphi}{r'} \right) = \frac{1}{r'^2} \frac{\partial}{\partial r'} \left[\mu r'^3 \frac{\partial}{\partial r'} \left(\frac{u'_\varphi}{r'} \right) \right], \tag{8}$$

$$\frac{5}{2} \frac{P}{T} u'_r \frac{\partial T}{\partial r'} - \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \kappa \frac{\partial T}{\partial r'} \right) - u'_r \frac{\partial P}{\partial r'} = \mu \left\{ \frac{4}{3} \left[\frac{1}{r'} \frac{\partial (r' u'_r)}{\partial r'} \right]^2 - 4 \frac{u'_r}{r'^2} \frac{\partial (r' u'_r)}{\partial r'} + 4 \frac{u'^2_r}{r'^2} + \left[r' \frac{\partial}{\partial r'} \left(\frac{u'_\varphi}{r'} \right) \right]^2 \right\}. \tag{9}$$

In the above equations the mass density ϱ has been represented by the equation of state $\varrho = mP/kT$. Eqs (7) - (9) refer to a system of four coupled differential equations for the fields $u'_r(r'), u'_\varphi(r'), P(r'), T(r')$ which we proceed to find its solution for given boundary conditions.

We note first that for a linearized problem, in which the conditions (1), (3) and (4) hold, we can write

$$P(r') = P_0(r') \left[1 + \sum_\alpha v^{(\alpha)} X_\alpha \right], \qquad u'_r(r') = \beta^{-1} \left[\sum_\alpha u_r^{(\alpha)} X_\alpha \right], \tag{10}$$

$$u'_\varphi(r') = \Omega_0 r' + \beta^{-1} \left[\sum_\alpha u_\varphi^{(\alpha)} X_\alpha \right], \qquad T(r') = T_0 \left[1 + \sum_\alpha \tau^{(\alpha)} X_\alpha \right], \tag{11}$$

where $\alpha = P, \Omega, T$, $v^{(\alpha)}, u_r^{(\alpha)}, u_\varphi^{(\alpha)}$ and $\tau^{(\alpha)}$ are dimensionless quantities and X_α are the thermodynamic forces $X_P = \Delta P/P_0(R_1)$, $X_\Omega = \beta R_1 \Delta \Omega$ and $X_T = \Delta T/T_0$.

Next we insert (10) and (11) into Eqs (7) - (9) and get the linearized field equations for the dimensionless quantities $v^{(\alpha)}, u_r^{(\alpha)}, u_\varphi^{(\alpha)}$ and $\tau^{(\alpha)}$ that read

$$\frac{d \ln[n_0 r u_r^{(\alpha)}]}{dr} = 0, \qquad \frac{dv^{(\alpha)}}{dr} + 2r\omega^2 \tau^{(\alpha)} = 4\omega u_\varphi^{(\alpha)} + \frac{16 n_{00}}{3 n_0} u_r^{(\alpha)} r^2 \frac{\omega^4}{\delta}, \tag{12}$$

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d(u_\varphi^{(\alpha)} r)}{dr} \right] = \frac{4 n_0}{n_{00}} u_r^{(\alpha)} \omega \delta, \qquad \frac{d}{dr} \left[r \frac{d\tau^{(\alpha)}}{dr} \right] = -\frac{16 n_{00}}{15 n_0} u_r^{(\alpha)} r^2 \omega^2 \delta, \tag{13}$$

where $r = r'/R_0$ is a dimensionless quantity. The general solution of the system (12) - (13) for the fields $v^{(\alpha)}, u_r^{(\alpha)}, u_\varphi^{(\alpha)}$ and $\tau^{(\alpha)}$ are

$$u_r^{(\alpha)} = \frac{D^{(\alpha)}}{n_0 r} n_{00}, \quad u_\varphi^{(\alpha)} = 2\delta\omega D^{(\alpha)} \left(r \ln r - B_1^{(\alpha)} r + \frac{B_2^{(\alpha)}}{r} \right), \quad \tau^{(\alpha)} = \frac{4}{15} D^{(\alpha)} \omega^2 \delta \left(C_1^{(\alpha)} \ln r + C_2^{(\alpha)} - r^2 \right), \tag{14}$$

$$v^{(\alpha)} = 8\delta\omega^2 D^{(\alpha)} \left[\frac{1}{2} g(r) - \frac{B_1^{(\alpha)}}{2} r^2 + B_2^{(\alpha)} \ln r + \frac{\omega^2}{60} \left(r^4 - 2C_2^{(\alpha)} r^2 - 2C_1^{(\alpha)} g(r) \right) + L^{(\alpha)} \right] - \frac{4}{3} \frac{n_{00}^2}{n_0^2(r)} \frac{D^{(\alpha)} \omega^2}{\delta}, \tag{15}$$

where $D^{(\alpha)}$, $B_1^{(\alpha)}, B_2^{(\alpha)}, C_1^{(\alpha)}, C_2^{(\alpha)}$, and $L^{(\alpha)}$ are constants of integration and $g(r) = r^2 \ln r - r^2/2$.

Once the fields $u_r^{(\alpha)}(r)$, $u_\varphi^{(\alpha)}(r)$, $\tau^{(\alpha)}(r)$, $\upsilon^{(\alpha)}(r)$ are known we can also obtain the non-vanishing components of the heat flux vector and for the viscous stress tensor from (6), (14) and (15):

$$q_r^{(\alpha)} = -\frac{15}{8}\frac{1}{\delta}\frac{d\tau^{(\alpha)}}{dr} = -\frac{1}{2}D^{(\alpha)}\omega^2\left(C_1^{(\alpha)}\frac{1}{r} - 2r\right), \quad \sigma_{rr}^{(\alpha)} = \frac{1}{\delta}\left[\frac{2}{3}\frac{du_r^{(\alpha)}}{dr} - \frac{1}{3}\frac{u_r^{(\alpha)}}{r}\right] = -\frac{1}{3}\frac{D^{(\alpha)}}{\delta r^2}\frac{n_{00}}{n_0(r)}\left(3 + 4\omega^2 r\right),$$

(16)

$$\sigma_{\varphi\varphi}^{(\alpha)} = \frac{1}{\delta}\left[\frac{2}{3}\frac{u_r^{(\alpha)}}{r} - \frac{1}{3}\frac{du_r^{(\alpha)}}{dr}\right] = \frac{1}{3}\frac{D^{(\alpha)}}{\delta r^2}\frac{n_{00}}{n_0(r)}\left(1 - 2\omega^2 r\right), \quad \sigma_{r\varphi}^{(\alpha)} = \frac{1}{2\delta}\left[\frac{du_\varphi^{(\alpha)}}{dr} - \frac{u_\varphi^{(\alpha)}}{r}\right] = D^{(\alpha)}\omega\left(1 - 2\frac{B_2^{(\alpha)}}{r^2}\right).$$

(17)

For $\omega=0$ the solution of the system of differential equations (12) and (13) reduces to

$$u_r^{(\alpha)}r = D_0^{(\alpha)}, \qquad u_\varphi^{(\alpha)} = E_1^{(\alpha)}r + \frac{E_2^{(\alpha)}}{r}, \qquad \tau^{(\alpha)} = F_1^{(\alpha)}\ln r + F_2^{(\alpha)}, \qquad \upsilon^{(\alpha)} = G^{(\alpha)},$$

(18)

where $D_0^{(\alpha)}$, $E_1^{(\alpha)}$, $E_2^{(\alpha)}$, $F_1^{(\alpha)}$, $F_2^{(\alpha)}$ and $G^{(\alpha)}$ are constants of integration.

III SLIP AND JUMP BOUNDARY CONDITIONS

In order to determine the constants of integration of the previous section one has to know the boundary conditions at the outer cylinder ($r = 1$) and at the inner cylinder ($r = R_1/R_0$). The slip and jump boundary conditions adopted here that correspond to a perturbation of the pressure, temperature and angular velocity on the outer cylinder are:

$$\upsilon^{(\alpha)} = \alpha_1 u_r^{(\alpha)} - \frac{\alpha_2}{\delta}\frac{n_{00}}{n_0(1)}\frac{d\tau^{(\alpha)}}{dr} - 2\frac{\alpha_3}{\delta}\frac{n_{00}}{n_0(1)}\frac{du_r^{(\alpha)}}{dr} - \frac{\alpha_4}{\delta}\frac{n_{00}}{n_0(1)}u_r^{(\alpha)},$$

(19)

$$\tau^{(\alpha)} = \alpha_5 u_r^{(\alpha)} - \frac{\alpha_6}{\delta}\frac{n_{00}}{n_0(1)}\frac{d\tau^{(\alpha)}}{dr} - 2\frac{\alpha_7}{\delta}\frac{n_{00}}{n_0(1)}\frac{du_r^{(\alpha)}}{dr} - \frac{\alpha_8}{\delta}\frac{n_{00}}{n_0(1)}u_r^{(\alpha)},$$

(20)

$$u_\varphi^{(\alpha)} = -\frac{\alpha_9}{\delta}\frac{n_{00}}{n_0(1)}\left(\frac{du_\varphi^{(\alpha)}}{dr} - \frac{u_\varphi^{(\alpha)}}{r}\right);$$

(21)

while the corresponding slip and jump boundary conditions on the inner cylinder read

$$\upsilon^{(\alpha)} = \delta_{P\alpha} - \alpha_1 u_r^{(\alpha)} + \frac{\alpha_2}{\delta}\frac{n_{00}}{n_0(R_1/R_0)}\frac{d\tau^{(\alpha)}}{dr} + 2\frac{\alpha_3}{\delta}\frac{n_{00}}{n_0(R_1/R_0)}\frac{du_r^{(\alpha)}}{dr} - 2\frac{\alpha_4}{\delta}\frac{n_{00}}{n_0(R_1/R_0)}u_r^{(\alpha)},$$

(22)

$$\tau^{(\alpha)} = \delta_{T\alpha} - \alpha_5 u_r^{(\alpha)} + \frac{\alpha_6}{\delta}\frac{n_{00}}{n_0(R_1/R_0)}\frac{d\tau^{(\alpha)}}{dr} + 2\frac{\alpha_7}{\delta}\frac{n_{00}}{n_0(R_1/R_0)}\frac{du_r^{(\alpha)}}{dr} - 2\frac{\alpha_8}{\delta}\frac{n_{00}}{n_0(R_1/R_0)}u_r^{(\alpha)},$$

(23)

$$u_\varphi^{(\alpha)} = \delta_{\Omega\alpha} + \frac{\alpha_9}{\delta}\frac{n_{00}}{n_0(R_1/R_0)}\left(\frac{du_\varphi^{(\alpha)}}{dr} - \frac{u_\varphi^{(\alpha)}}{r}\right).$$

(24)

In the above equations $\delta_{P\alpha}, \delta_{T\alpha}$ and $\delta_{\Omega\alpha}$ are Kronecker symbols and $\alpha = P, T, \Omega$.

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The terms in the jump of the pressure as stated in the boundary conditions above have the following meaning: (a) $\alpha_1 u_r^{(\alpha)}$ is due to the evaporation and condensation at the solid boundary; (b) $(\alpha_2/\delta)(d\tau^{(\alpha)}/dr)$ is due to a normal gradient of temperature at the solid boundary; (c) $(2\alpha_3/\delta)(du_r^{(\alpha)}/dr)$ is due to a normal gradient of the normal component of the velocity at the solid boundary; (d) $(\alpha_4/\delta)u_r^{(\alpha)}$ and $(2\alpha_4/\delta)u_r^{(\alpha)}$ are due to the curvature of the solid surfaces. The modulus of the mean curvature, as defined in the work by Sone [10], is equal to one for the internal cylinder and 1/2 for the external cylinder. The terms in the jump of the temperature have the same meaning as those that appear in the jump of the pressure. The jump in the tangential velocity is due to the normal gradient of the tangential velocity. The coefficients α_1 through α_9 are calculated from a kinetic equation and the values for these coefficients, based on the BGK model, are presented by Sone [10]. Since we have applied the S model to solve the present problem in the kinetic regime [9], we have to use the coefficients based on the S model too. It was verified that the coefficients α_2 and α_6 based on the S model are 3/2 times greater than those based on the BGK model, while the other coefficients are exactly the same. Thus, the numerical data presented by Sone [10] can be used here if the two above mentioned coefficients are multiplied by 3/2. We have: $\alpha_1 = 2.13204$, $\alpha_2 = 0.83766$, $\alpha_3 = 0.82085$, $\alpha_4 = 0.38057$, $\alpha_5 = 0.44675$, $\alpha_6 = 1.95408$, $\alpha_7 = 0.33034$, $\alpha_8 = 0.13157$ and $\alpha_9 = 1.01619$.

Once the boundary conditions are known it is easy to obtain the constants of integration which appear in the hydrodynamic solutions (14), (15) and (18). Hence the fields of density, velocity, temperature, heat flux vector and stress tensor can be determined as functions of the rarefied parameter and of the angular velocity.

IV RESULTS

For the flow caused by a pressure difference we have plotted in Figures 1 to 3 the kinetic and the hydrodynamic solutions of the fields $u_r^{(P)}$, $q_r^{(P)}$ and $\sigma_{r\varphi}^{(P)}$ as functions of the rarefied parameter δ for different values of the angular velocity ω. We note that a better agreement of the hydrodynamic and kinetic solutions for $u_r^{(P)}$ and $q_r^{(P)}$ is attained for low angular velocities. The difference in the hydrodynamic and kinetic solutions for $\sigma_{r\varphi}^{(P)}$ is due to the fact that it is small as any cross-effect. The kinetic and the hydrodynamic solutions of the fields $u_r^{(\Omega)}$, $q_r^{(\Omega)}$ and $\sigma_{r\varphi}^{(\Omega)}$ for a flow caused by the velocity difference are plotted in Figures 4 to 6 as functions of the rarefied parameter δ and of the angular velocity ω. While there is a good agreement of the hydrodynamic and kinetic solutions for $\sigma_{r\varphi}^{(\Omega)}$, the difference between the solutions for $u_r^{(\Omega)}$ and $q_r^{(\Omega)}$ is due to the fact that both represent fields of cross-effects, which are usually small. The fields $u_r^{(T)}$, $q_r^{(T)}$ and $\sigma_{r\varphi}^{(T)}$ which are solutions of the hydrodynamic and kinetic equations are plotted in Figures 7 to 9 for a flow caused by the temperature difference as functions of the rarefied parameter δ and of the angular velocity ω. There is a very good agreement of the hydrodynamic and kinetic solutions for $\sigma_{r\varphi}^{(T)}$ and $q_r^{(T)}$ and we note that for high rarefaction parameter the heat flux $q_r^{(T)}$ does not depend on ω and all curves coincide. The field $u_r^{(T)}$ corresponds also to a cross-effect but here the agreement between the solutions is better that the two cases analyzed above.

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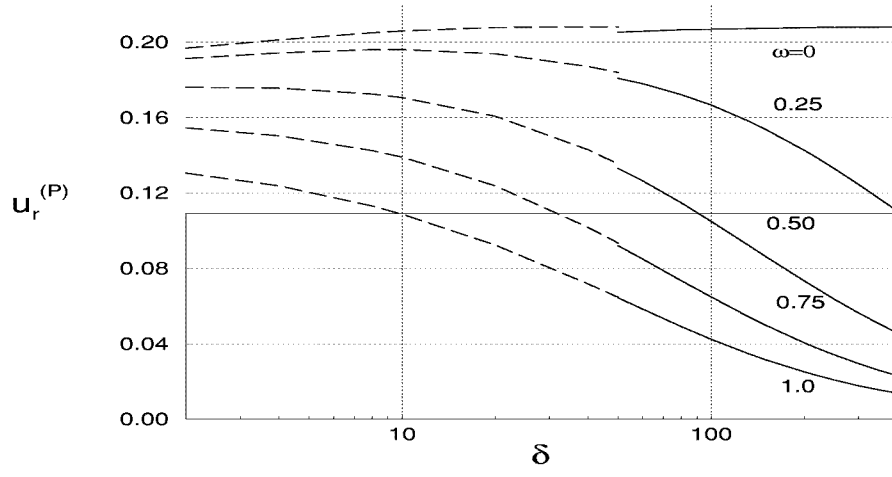
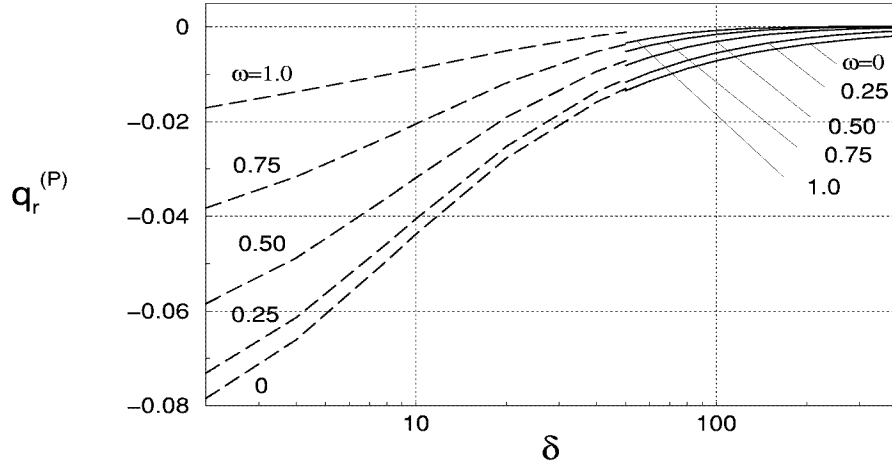**FIGURE 1.** $u_r^{(P)}$ as function of δ for different values of ω.

FIGURE 2. $q_r^{(P)}$ as function of δ for different values of ω.
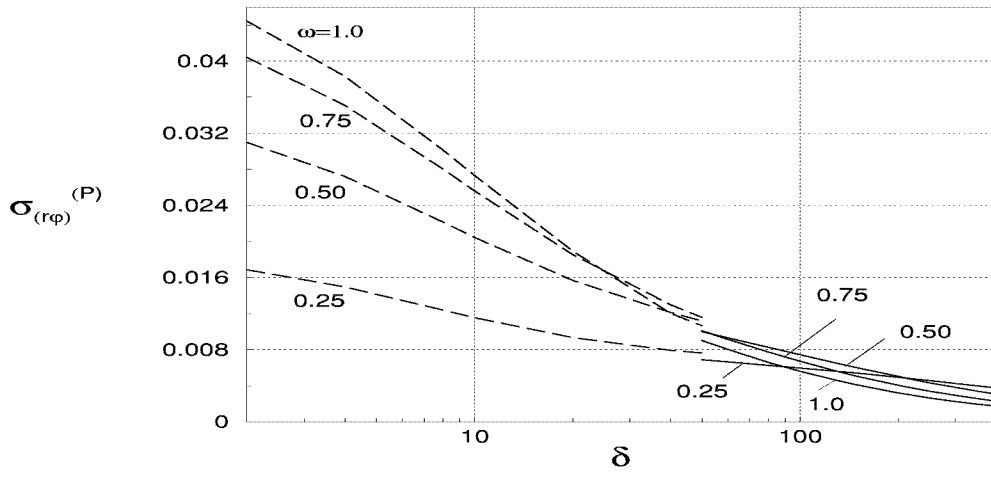
FIGURE 3. $\sigma_{r\varphi}^{(P)}$ as function of δ for different values of ω.
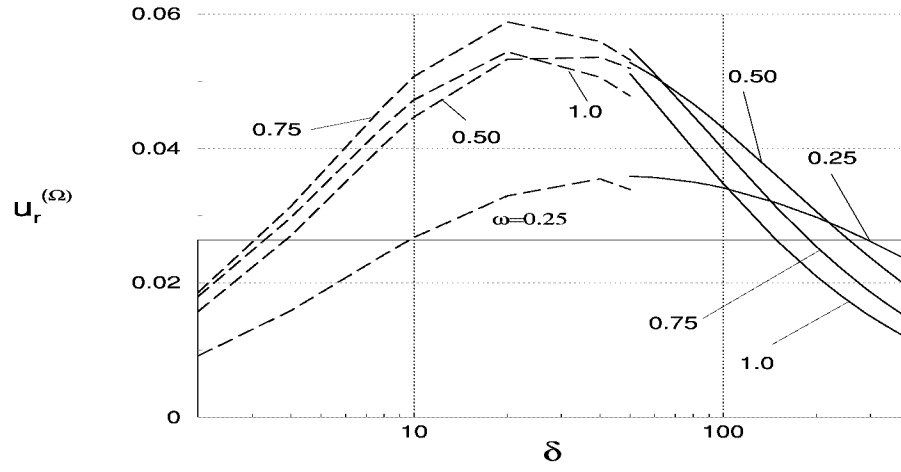
FIGURE 4. $u_r^{(\Omega)}$ as function of δ for different values of ω.
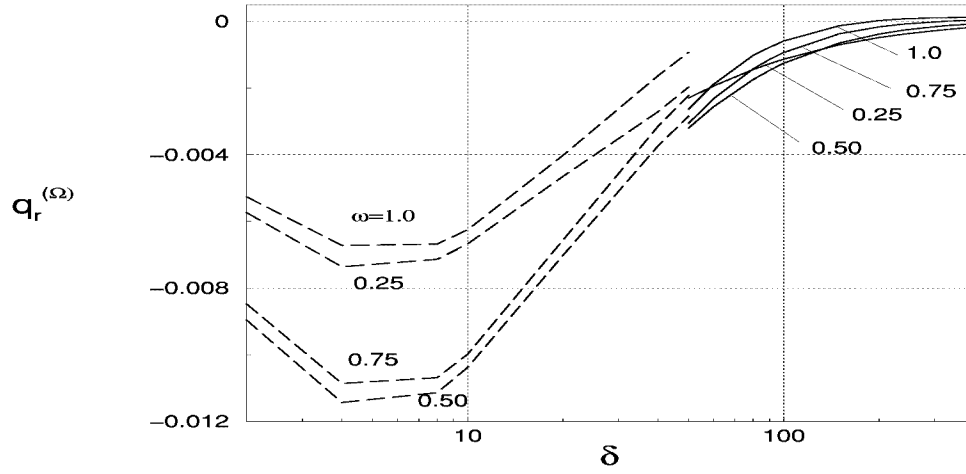
FIGURE 5. $q_r^{(\Omega)}$ as function of δ for different values of ω.
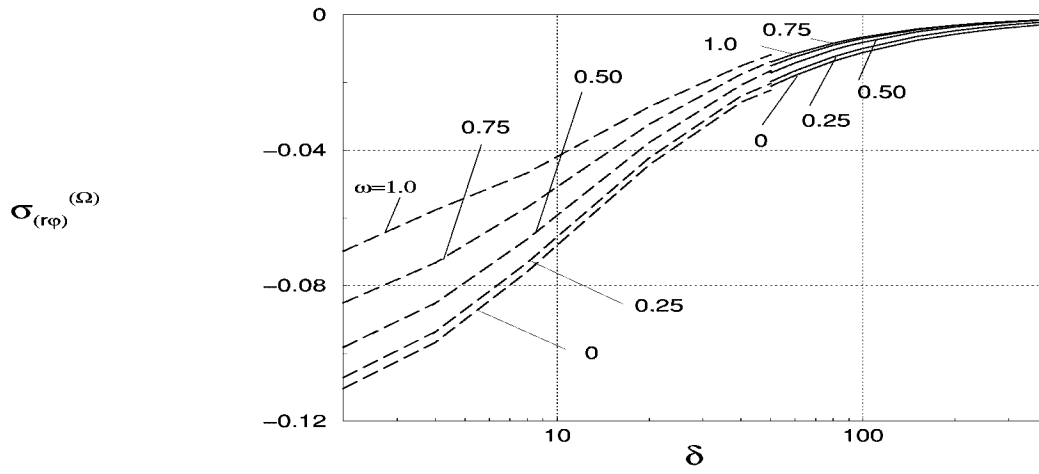
FIGURE 6. $\sigma_{r\varphi}^{(\Omega)}$ as function of δ for different values of ω.
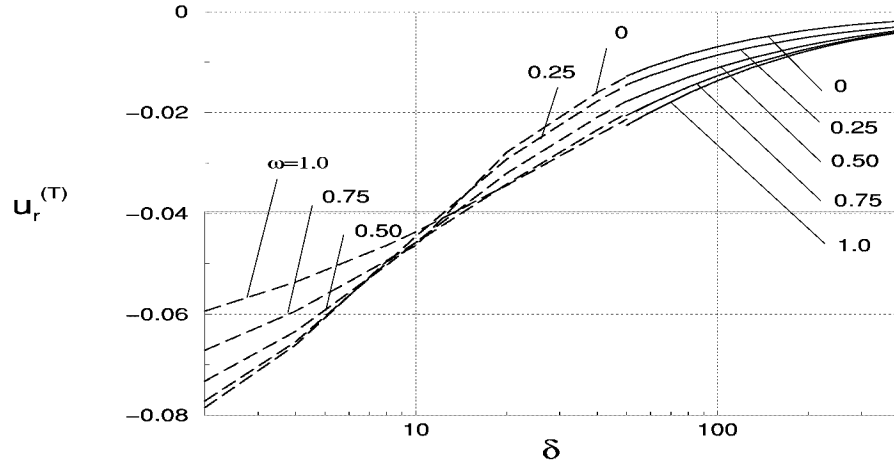
FIGURE 7. $u_r^{(T)}$ as function of δ for different values of ω.

FIGURE 8. $q_r^{(T)}$ as function of δ for different values of ω.
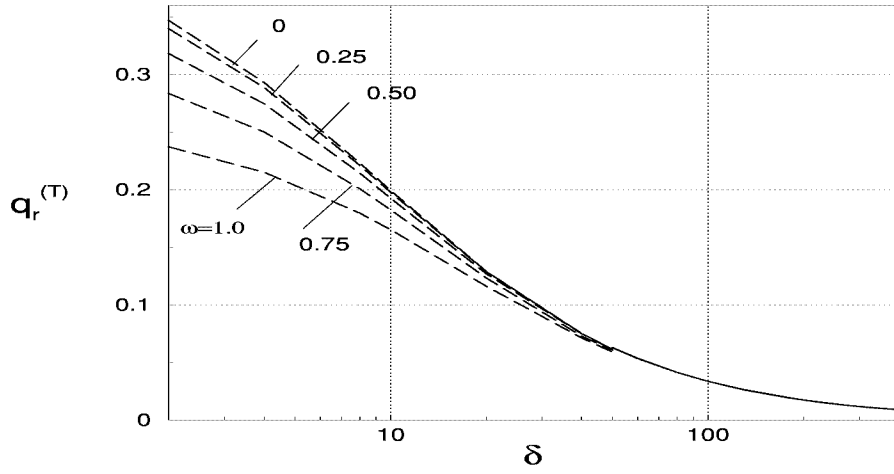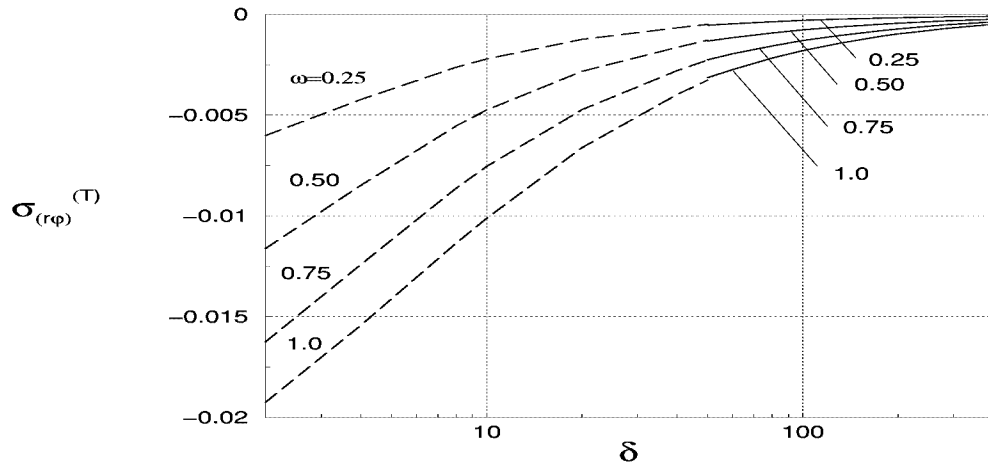

FIGURE 9. $\sigma_{r\varphi}^{(T)}$ as function of δ for different values of ω.